

Philadelphia Classic

The Dining Philosophers
Department of Computer and Information Science
School of Engineering and Applied Science
University of Pennsylvania
dp-exec@googlegroups.com
<http://dp.seas.upenn.edu>

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1. Guest Seating on the Titanic: Counting Partitions

For the upcoming voyage of the Titanic, the Captain informed his staff members that he wanted a group of V.I.P.s to be seated in two, three, or possibly even four sections for the formal dinner. You have been asked to develop a series of seating charts that show all the possible ways that the V.I.P.s could be seated. To do this, you will first need to determine how many charts will need to be created so that you can order the correct number of poster boards to display these charts. Being a computer scientist, you realize that the group of V.I.P.s is a *set* - namely a collection of distinct objects, and therefore this set can be broken up into non-empty subsets called *partitions*.

For example, given a set of 3 people: $\{James, Leonardo, Kate\}$ there are 3 possible ways to divide these elements into two partitions (labeled A and B). This is how it the divisions might appear:

	Partition A	Partition B
Possibility 1:	$\{James, Leonardo\}$	$\{Kate\}$
Possibility 2:	$\{James, Kate\}$	$\{Leonardo\}$
Possibility 3:	$\{James\}$	$\{Leonardo, Kate\}$

This illustrates every possible way that the set of 3 people can be divided into 2 partitions. Therefore, the total number of ways that a 3-element set can be partitioned into 2 parts is 3. The formula to calculate this number is called the Stirling Numbers of the Second Kind and is commonly written $S(n, k)$ where n is the number of elements in the set that will be partitioned and k is the number of desired partitions. This formula can be expressed in the following recursive relationship:

$$S(n, k) = S(n - 1, k - 1) + kS(n - 1, k)$$

The recursion terminates when either $k = 1$ or $n = k$ and in either case $S(n, n) = S(n, 1) = 1$. An illustration of the Stirling Numbers formula for a 3 element set with 2 partitions is as follows:

$$\begin{aligned} S(3, 2) &= S(2, 1) + 2 \cdot S(2, 2) \\ S(3, 2) &= 1 + 2 \cdot 1 = 3 \end{aligned}$$

Your task is to implement a program to calculate Stirling Numbers of the Second Kind. This program should take two integers as input (separated by a space) and return the number of possible ways to partition the set given the input parameters corresponding to n and k .

Examples

Input: 3 2
Output: 3

Input: 8 3
Output: 966

2. Squares

Jack and Rose decide to join their pals in steerage for a dance. Having heard from Jack about the outrageous fun the lower classes have every night, Rose is glad to get away for the evening.

Upon entering the room, Jack and Rose witness the crowd performing a unique form of line dance. The room is filled with appear to be concentric circles. Upon closer examination, however, it appears that this tiny open space has forced the dancers into patterns of squares.

Jack and Rose, seeking to join the dance, need to determine the overall lay of the room. At the beginning of each dance, the fiddle player calls out a number, and the crowd forms into concentric squares of that number.

Your task is to take as input the number called, and return a printout (to screen) of a top-down view of the dance floor. The innermost square is always a single person. There needs to be a person's width between adjacent people. You can use the space character to represent space. See the samples below.

Examples

Input: 2

Output:

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X X X
```

```
X X X
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```
X X X
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Input: 3

Output:

```
X X X X X
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3. Matrix Determinants

Unbeknownst to historians, the Titanic was equipped with a prototype resonance ray capable of destroying any hazardous obstacles the ship encountered on its voyages. Properly tuned, the weapon could cause the target object's molecules to vibrate at their natural frequency, tearing it apart. Unfortunately, when the engineers aboard the Titanic saw the iceberg, they could not find its the natural frequency and use the resonance ray to save everyone because they did not know how to compute matrix determinants; frequencies can be written in terms of eigenvalues, which are derivable from the determinant.

A matrix determinant can be calculated through a recursive process known as cofactor expansion. The base case for the calculation is a 2×2 matrix:

$$\det \begin{bmatrix} a & b \\ c & d \end{bmatrix} = ad - bc$$

The general case is a little bit harder and is best described by an example:

$$\det \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} = (-1)^{0+0}a \cdot \det \begin{bmatrix} e & f \\ i & j \end{bmatrix} + (-1)^{0+1}b \cdot \det \begin{bmatrix} e & f \\ g & i \end{bmatrix} + (-1)^{1+1}c \cdot \det \begin{bmatrix} d & e \\ g & h \end{bmatrix}$$

First, choose a row to expand (usually the first one). Then for each element in the row, take its product with $(-1)^{k+l}$ (where k and l are the row and column, respectively, of the current element) and multiply the result by the determinant of the smaller matrix you obtain by deleting row k and column l from the original, larger matrix. Doing this for all elements in the row and then summing the results yields the determinant.

Input: Square matrix in the form of [a b c; d e f; g h i] where [denotes the start of a matrix,] denotes the end, ; represents a new row, and spaces separate elements. Be sure to handle negative numbers, but you do not have to handle any double digit numbers (i.e. any numbers ≥ 9).

Output: Single number representing the determinant of the input matrix.

Examples

Input: [2 2; 3 4]

Output: 2

Input: [2 1 2; 3 3 3; 1 4 2]

Output: 3

4. The Shamwow! Paradox

A young inventor from Germany named Vinzenz Offer was also aboard the Titanic for its fateful voyage. When he learned that the Titanic had sprung a leak, he dug into his suitcase and found his latest creation, an absorbent towel that could hold a quantity of liquid proportional to its surface area. He realized that by cutting the towel in half some number of times it could absorb an arbitrarily large amount of water, thereby saving him from drowning.

The starting dimensions of this towel (in cm) are given as the first three inputs: length, then width, then depth.

The ratio of square centimeters of exposed area to mL absorbency is the fourth input. (for example, 2 would indicate that for every 1 square centimeter exposed, the towel can absorb 2 mL of water)

The amount of water (in liters) needed to be absorbed is given as the fifth input. At each step, Vinzenz cuts every piece of the towel in half, doubling its surface area, and places it in front of his cabin door to stem the flow of water. (That is, Step 0 has 1 piece, Step 1 has 2 pieces, Step 3 has 4 pieces, and so on.)

How many steps must occur to absorb the desired amount of liquid?

Assume Vinzenz cuts along the depth edge every time and thus that the length and width will be identical across all the pieces. Also, all inputs will be integers.

Examples

Input: 9 9 2 1 2

Output: 4

Input: 6 6 4 2 4

Output: 5

5. Settling the Titanic's blame: Game of War

In the final minutes of the Titanic's fateful voyage, the captain and the chief officer settled who should take the blame for the ship's woes. There was no civilized way to determine responsibility so they turned to their favorite card game: War. You have been asked to create a simulator for a modified version of this game.

The deck only includes 10 cards: 1 through 5 (inclusive) with 2 of each.

A deck is divided into two halves (5 cards for each of the two players). You will be provided with an ordered list of both player's starting decks.

The first list represents Player1's starting deck and the second list is Player2's. As a convention, first number in the list will represent the top of the face down deck and the last number will be the bottom.

Game Play: Each round, Player1 and Player2 each draw their top card (beginning of the list) from their deck.

There are 3 possible scenarios:

- Player1 plays a card higher than Player2
Player1's card is placed facedown on top of Player2's card. The two cards are placed on the bottom of Player1's deck (end of the list).
- Player2 plays a card higher than Player1
Player1's card is placed facedown on top of Player2's card. The two cards are placed on the bottom of Player2's deck (end of the list).
- Player1 plays the same card as Player2
Player1's card is placed facedown on top of Player2's card and held to the side. They each draw again. If Player1 or Player2 wins: Player1's card is placed on top of Player2's card. These cards are placed under the cards put the side and placed a the end of the deck. Repeat if needed.

The game ends when a player holds all cards. This player is declared the winner. You should return the deck of the winning player.

Examples

Input: [1, 2, 2, 3, 3], [1, 4, 4, 5, 5]

Output: [1, 1, 2, 4, 2, 4, 3, 5, 3, 5]

Input: [5, 1, 3, 2, 5], [4, 2, 3, 1, 4]

Output: [3, 3, 2, 1, 5, 4, 5, 1, 4, 2]

6. Golf balls on the deck of the Titanic

Cal Hockley is working on his drive on the deck of the Titanic, and orders a servant to bring him some golf balls. While waiting for the servant to return, Cal determines that the most efficient way to prepare the golf balls is to stack them in a pyramid-like structure such that every ball (with the exception of the bottom layer) must rest on top of 4 other balls.

Given the number of golf balls recieved, how should Cal stack the golf balls?

(Note: The base of the golf ball pyramid structure does not have to be a perfect square, but each layer must be a rectangle and no golf balls may go unused.)

Input: The number of golf balls.

Output: The height of the pyramid.

Examples

Input: 14

Output: 3 (the sizes of the layers are 9 balls, 4 balls, and 1 ball)

Input: 60

Output: 4 (the sizes of the layers are 28 balls, 18 balls, 10 balls, and 1 ball)

7. Lifeboat Rescue

As rescue ships began to arrive, one spotted a drifting lifeboat and the crew needed to figure out if they could reach them on their present course. The crew of the rescue ship know their own location, direction and speed and they can compute those of the lifeboat. The ship happened to also have one of those newfangled mechanical computing devices on it and this is the perfect task for it! As the technology expert aboard, your job is to write an algorithm that, given the positions, directions and velocities of the rescue ship and the lifeboat, will determine if you can reach the lifeboat in time (intercept its path) or must change course. You will be provided with data in the following format x coordinate of ship, y coordinate of ship, slope of ship's path and ship's velocity followed by the same parameters for the lifeboat. Your output should say "Yes" if the ship can intercept the lifeboat in time and "No" if they need to change course to do so.

Examples

Input: 0 0 1 3 1 0 2 1 //Ship x , Ship y , Ship slope, Ship vel, Boat x , Boat y , Boat slope, Boat vel

Output: Yes

Input: 0 1 0 1 0 0.5 2 10

Output: No

8. Set operations aboard the Titanic

The Titanic's Captain, in his numerous responsibilities, is required to be able to determine who will be in attendance for various ship functions. The Captain possesses lists of all ship personnel, lists of staff by department, and lists of passengers by deck - each of which can be interpreted as a *set*. To make sure that the people aboard are properly accommodated, he will need to combine these sets in different ways. For example, to create a set of all people requiring dinners aboard the ship, the Captain will need to build a set of people based on the union of the passenger and crew sets.

The Captain has assigned you the task of building a program that will take two sets as input, and generate the Union, Intersection and Difference of the two sets. The following are definitions for the Union, Intersection and Difference operators as they apply to two sets:

$$\text{Union Operator: } A \cup B \triangleq \{\omega : \omega \in A \text{ or } \omega \in B\}$$

This expression is read "The Union of two sets A and B is defined to be the set of all elements ω where ω is an element of A or ω is an element of B."

$$\text{Intersection Operator: } A \cap B \triangleq \{\omega : \omega \in A \text{ and } \omega \in B\}$$

This expression is read "The Intersection of two sets A and B is defined to be the set of all elements ω where ω is an element of A and ω is an element of B."

$$\text{Difference Operator: } A \setminus B \triangleq \{\omega : \omega \in A \text{ and } \omega \notin B\}$$

This expression is read "The Difference of two sets A and B is defined to be the set of all elements ω where ω is an element of A and ω is NOT an element of B."

Your program should accept two sets, each delimited by curly braces. Within each set, the elements should be separated by commas and can be any string of letters or numbers including spaces. Your program should then print out the Union, Intersection and Difference of the two sets.

Examples

Input: {Jack,Rose,Molly}{Brock,Rose,Ruth}

Output:

Union: {Jack,Rose,Molly,Brock,Ruth}

Intersection: {Rose}

Difference: {Jack,Molly}

Input: {Caledon,Bobby}{Bruce,Spicer}

Output:

Union: {Caledon,Bobby,Bruce,Spicer}

Intersection: {}

Difference: {Caledon,Bobby}